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FIRST ORDER STATISTICAL PROPERTIES OF SIMULATED SPECKLE FIELDS.

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The study of random processes is a topic of major importance in a wide range of different problems. For example, it arises in optics in the study of speckle fields. These fields are produced when laser light is scattered from an optically rough surface. It is important to be able to model the propagation of such fields numerically, however the calculation may change the statistical properties of the propagated field. The first order statistics for the intensity of such fields follow a negative exponential distribution [1], i.e.

$$p_I(I) = \frac{1}{\tilde{I}} \exp\left(-\frac{I}{\tilde{I}}\right), \tag{1}$$

where I is the recorded intensity and $\tilde{I}$ is the mean intensity, while the phase is uniformly distributed between 0 and 2 π. In the paraxial regime one may relate the fields at two different planes using the Fresnel transform. Often however this transform is not amenable to an analytic solution and so has to be calculated numerically, according to Eq. (2)

$$u(x) = \frac{\delta}{\sqrt{j\lambda z}} \sum_{n=1}^{N} U_n \exp\left[\frac{j\pi\left(x - X_n\right)^2}{\lambda z}\right], \tag{2}$$

where z is the distance between planes, λ the wavelength of the light, x is the spatial variable in the output domain, δ is the distance between adjacent sampled points, U_n is input field sampled at N discrete locations indicated by the spatial vector X_n. In Fig. 1, we present a plot of $|u(x)|^2$ for the following values: λ = 633 nm, z = 25 cm, X_n spans 2 mm. The numerical computation generates a series of replicas in the output plane separated from each other by $(\lambda z)/\delta$, one of which (blue distribution) can be seen in Fig. 1. In order to ensure the replicas are well separated from each other as in Fig. 1, we need to carefully choose U_n. In this example, N/2 complex values (where the real and imaginary values were uniformly distributed between -1 and +1) were interpolated to N samples by zero-padding in the Fourier domain giving the input matrix U_n. A histogram of the resulting intensity values is shown in Fig. 2. These calculations were run for many different initial random inputs and this is a representative example. Importantly the numerically calculated intensity distribution still follows Eq. (1) and hence seems to be a plausible physcial model. To find the probability that an intensity value lies in the first "bin" in Fig. 2, we integrate $p(I)$ over $0 < I < 0.01$, to give 0.316. From the law of large numbers, $p(I) \sim 3336/10000 \sim 0.3336$ in keeping with Eq. (1). This result has significant implications for using the spectral method for propagating over short distances.

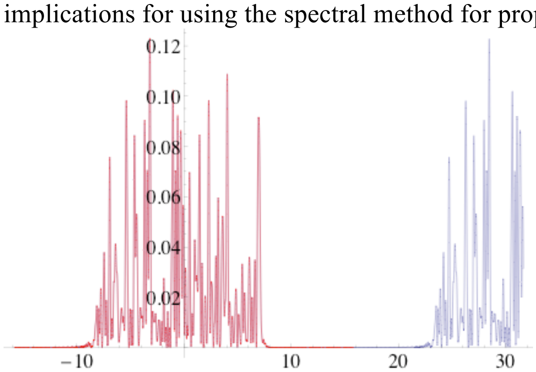

Figure 1. Numerical results for the diffraction of a speckle field, intensity distribution is plotted, axes in mm.

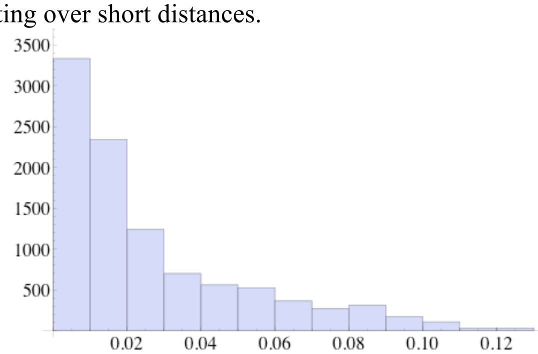

Figure 2. Histogram of 10,000 intensity values, which follow a negative exponential distribution.

REFERENCES

[1] J. W. Goodman, *Speckle Phenomena in Optics: Theory and Applications*, 1st ed (Roberts, 2007).